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THE ASYMPTOTIC LAWS OF SHOCKLESS STRONG COMPRESSION OF QUASI-ONE-DIMENSIONAL GAS LAYERS†

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The solutions of initial-boundary-value problems describing the shockless compression of cylindrically and spherically symmetric layers on an ideal polytropic gas to infinite density are investigated. Attention is also devoted to the quasi-one-dimensional case, when the surface on which the compression takes place is in one-to-one correspondence with the sonic characteristic surface separating the initial background flow and the compression wave. The solutions are expanded in convergent power series in a space of special dependent and independent variables, both in the neighbourhood of the final time. Asymptotic laws of shockless strong compression are found, and it is proved that they are described by curves in the convergence domains of the series. The additional external energy resources required for the transition from the compression of plane layers to that of quasi-one-dimensional layers are shown to be finite, provided that the polytropy index of the gas is not greater than three.

A mathematical description of the process of shockless isentropic compression of an ideal gas to any prescribed density value, including infinity (for a detailed bibliography see [10]), is of interest in relation to the problem of laser thermonuclear fusion [2, 3]. In the case of plane-symmetric flows, a simple centred Riemann wave describes the compression of a plane gas layer to infinite density [4]. By matching a centred Riemann wave with a uniform gas flow one can obtain any finite value of the density in a compressed plane layer [5]. In the case of cylindrically and spherically symmetric flows, Sedov's self-similar solutions [6] describe the shockless strong compression of an ideal gas which is initially homogeneous and at rest inside a cylinder or a sphere (see, e.g. [7, 8]).

As to the choice of optimal laws of motion for impermeable pistons, which give shockless compression of one-dimensional gas layers, it has been suggested that the scheme of piston motion proposed in [9], which identical with that presented earlier in [4, 5] (for compression to infinite or finite density, respectively), should be replaced by an essentially different scheme of piston motion [10] in which the point at which the compression wave is centred is on the piston. Then the resultant flow has not only the prescribed constant density but also zero gas velocity. Thus, a mode of shockless compression of a gas has been described [10] in which the work performed is expended entirely in compression, no further work being needed to increase the kinetic energy of the gas. Different exact solutions describing the unrestricted compression of special volumes of gas yield different asymptotic laws (see, e.g. [11]).

The aim of this paper is to derive and justify, on the basis of the mathematical theory developed in [1], more precise asymptotic laws of motion for impermeable pistons to achieve shockless strong compression of quasi-one-dimensional layers of an ideal gas. The study will be carried out in detail for the example of cylindrically and spherically symmetric gas layers.

1. CONSTRUCTION OF THE GAS FLOW

Consider isentropic uniform potential flows of an ideal polytropic gas described by the equation [12]

$$\Phi_{tt} + 2\Phi_{tr}\Phi_r + (\Phi_r^2 - c^2)\Phi_{rr} - \nu c^2 r^{-1}\Phi_r = 0$$

$$c^2 = (\gamma-1)\left(K - \Phi_t - \frac{1}{2}\Phi_r^2\right), \quad K = \text{const}, \quad r = (x_1^2 + \ldots + x_{\nu+1}^2)^{1/2}$$

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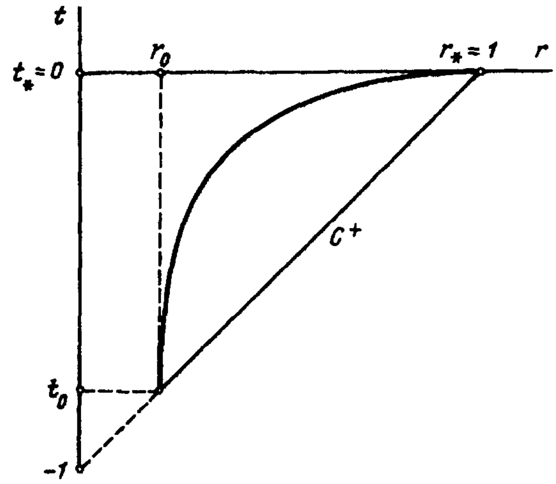

Fig. 1.

where $\nu = 0, 1, 2$ represent plane, cylindrical and spherical symmetry, respectively, $\Phi(t, r)$ is the flow potential, x_i $(i = 1, 2, 3)$ are Cartesian coordinates, $u = \Phi_r(t, r)$ is the gas velocity, c^2 is the square of the velocity of sound in the gas and is $\gamma = \text{const} > 1$ is the polytropy index of the gas.

Suppose we are seeking the flow produced when a cylindrical or spherical layer of gas is compressed by an impermeable piston (see Fig. 1). Without loss of generality, we may assume that the outer radius r_* of the initial gas layer is 1 and that the density of the gas and the velocity of sound in it for $0 < r_0 \leqslant r \leqslant r_*$ at time $t = t_0$ are also equal to 1. It is required to find both the law of motion of the piston (whose path is represented in the figure by the solid curve) and the entire gas flow between the piston and the sonic line C^+, which at time $t = t_* > t_0$ describes shockless compression of the gas layer to infinite density. We may assume without loss of generality that $t_* = 0$.

The flow being sought, considered in physical space, has a singularity at a time $t = 0$. To remove this singularity we introduce a new unknown function $\Psi(t, u) = ur - \Phi(t, r) + Kt$ and take t, u as independent variables. Then $\Psi(t, u)$ is a solution of the following characteristic Cauchy problem [1]

$$\Psi_{tt}\Psi_{uu} - \Psi_{tu}^2 + 2u\Psi_{tu} - u^2 + c^2 + \nu uc^2\Psi_{uu}\Psi_u^{-1} = 0$$

$$\Psi(t, 0) = \frac{t+1}{\gamma-1}, \quad \Psi_u(t, 0) = t+1, \quad \Psi_{uu}(0, u) = 0 \tag{1.1}$$

$$c^2 = (\gamma - 1)(\Psi_t - u^2/2)$$

To transform to physical space, we use the relation

$$r = \Psi_u(t, u) \tag{1.2}$$

The Jacobian $J = -\Psi_{uu}$ of the transformation for problem (1.1) vanishes at $t = 0$ and is non-zero in some neighbourhood of that time. A solution of the problem exists in t, u space, is unique and has no singularities in the neighbourhood of the point $t = 0$, $u = 0$. The solution of problem (1.1) has been used to describe the collapse of a one-dimensional cavity in a gas [3]. In the case of plane symmetry, the solution of problem (1.1) is a simple centred Riemann wave, written as follows [1, 14]:

$$\Psi(t, u) = \frac{t+1}{\gamma-1} + (t+1)u + \frac{\gamma+1}{4}tu^2$$

In the cases of cylindrical and spherical symmetry, the solution of problem (1.1) is expressed as convergent power series.

To investigate the properties of solutions of problem (1.1) in the neighbourhood of the point $t = 0$, it is convenient to use the following series [1, 13] (throughout, if summation indices are not shown, summation is performed from $k = 0$ to $k = \infty$)

$$\Psi(t,u)=\sum T_k(u)t^k/k! \tag{1.3}$$

where

$$\begin{aligned}
&T_0(u)=\frac{1}{\gamma-1}+u, \quad T_1(u)=\frac{1}{\gamma-1}v^2+\frac{1}{2}u^2\\
&T_2(u)=\nu\frac{\gamma+1}{(\gamma-1)^2}\left[\frac{v^{\alpha+1}}{(2-\alpha)(1-\alpha)}+\frac{v^3}{2-\alpha}-\frac{v^2}{1-\alpha}\right], \text{ if } \gamma\neq\frac{5}{3},\ \gamma\neq 3\\
&T_2(u)=6\nu v^2\left(v\ \ln v-\frac{u}{3}\right), \text{ if } \gamma=\frac{5}{3}\\
&T_2(u)=\nu v^2(u-\ln v), \text{ if } \gamma=3\\
&v=1+\frac{\gamma-1}{2}u, \quad \alpha=\frac{\gamma+1}{2(\gamma-1)}
\end{aligned} \tag{1.4}$$

For $k \geqslant 2$

$$T_{k+1}=\nu P_{k+1}(v, v^{\alpha}, v^{-1}, \ln v)$$

are polynomials in the arguments shown, and the highest power of v in T_{k+1} is $\max\{k+2, \alpha k+1\}$. Note that if $1<\gamma<3$, then $\alpha>1$.

The structure thus established for the coefficients of series (1.3) has enabled us to prove [1] that the convergence domain with respect to u is unbounded: a constant $M>0$ exists such that, for $u \geqslant 0$, hence also as $u \to +\infty$, the convergence domain of series (1.3) is at least

$$u|t|^{1/\beta}<M; \quad \beta=\alpha \ \text{ for } \ 1<\gamma<3, \quad \beta=1 \ \text{ for } \gamma\geqslant 3 \tag{1.5}$$

To investigate the properties of solutions of problem (1.1) in the neighbourhood of C^+: $r=t+1$, hence also for $t=t_0$, it is convenient to use another representation [1, 14]

$$\begin{aligned}
&\Psi(t,u)=(t+1)\left[\frac{1}{\gamma-1}+u+u^2Y(t,u)\right]\\
&Y(t,u)=\sum Y_k(t)u^k/k!
\end{aligned} \tag{1.6}$$

For $\nu=1$

$$Y_0(t)=\frac{\gamma+1}{2}\left(1-\frac{1}{\sqrt{t+1}}\right) \tag{1.7}$$

$$\begin{aligned}
&Y_k(t)=\sum_{i,j,l,m}\left[a_{kij}(t+1)^{-i/2}\ln^j(t+1)+b_{klm}(t+1)^{l/2}\ln^m(t+1)\right]\\
&k\geqslant 1, \ 0\leqslant i,j,m,l+m\leqslant k+1, \ 0\leqslant l\leqslant k-1
\end{aligned}$$

The convergence domain of series (1.6) for $\nu=1$, $u\geqslant 0$, $-1<t\leqslant 0$ is at least [1, 14]

$$M_1\frac{u}{\sqrt{t+1}}<1, \quad M_1=\text{const}>0 \tag{1.8}$$

For $\nu=2$

$$\begin{aligned}
&Y_0(t)=\frac{\gamma+1}{4}\ln(t+1)\\
&Y_k=\sum_{i,j}e_{kij}(t+1)^i\ln^j(t+1)\\
&k\geqslant 1, \ 0\leqslant i\leqslant k, \ 0\leqslant j, i+j\leqslant k+1
\end{aligned} \tag{1.9}$$

The convergence domain of series (1.6) for $\nu = 2, u \geqslant 0, -1 < t \leqslant 0$ is at least [1, 14]

$$M_2\zeta u < 1, \quad \zeta = \max\{1, |\ln(t+1)|\}, \quad M_2 = \text{const} > 0 \tag{1.10}$$

We will briefly consider the construction of the flow in the problem of the shockless strong compression of a gas in a quasi-one-dimensional layer—the surface Γ in (x_1, x_2, x_3) space on which the compression occurs at time $t = 0$ is in one-to-one correspondence with the sonic characteristic surface, which separates the initial background flow and the compression wave at time $t_0 \leqslant t \leqslant 0$. To simplify the discussion, we will assume that the initial background is a uniform gas at rest. When constructing [1] the required compression wave, one first transforms from the variables x_1, x_2, x_3 to the variables η, ξ_1, ξ_2, where η is the distance from an arbitrary point in space to the surface Γ and ξ_1 and ξ_2 are independent variables in terms of which Γ is given parametrically. The components of the velocity vector of the gas, u^1, u^2 and u^3 will be its projections onto the axes η, ξ_1 and ξ_2, respectively. As independent variables we then take t, c, ξ_1, ξ_2, and as the vector of unknown functions we take $\mathbf{U} = \{\eta, u^1, u^2, u^3\}$. The series

$$\mathbf{U} = \sum \mathbf{U}_k(c, \xi_1, \xi_2)t^k / k! \tag{1.11}$$

which constitute a solution of the problem of shockless strong compression are uniquely constructed and converge as $c \to +\infty$ in the domain [1]

$$c|t|^{1/\beta} < M(\xi_1, \xi_2), \quad M(\xi_1, \xi_2) > 0 \tag{1.12}$$

The first coefficients of the series, say for $\gamma \neq 5/3$ and $\gamma \neq 3$, are

$$\begin{aligned} u_0^1 &= \frac{2}{\gamma-1}(c-1) \\ u_1^1 &= M_1(\xi_1,\xi_2)c + M_2(\xi_1,\xi_2)c^2 + M_3(\xi_1,\xi_2)c^\alpha \\ \eta_0 &= 0, \quad \eta_1 = \frac{\gamma+1}{\gamma-1}\left(c - \frac{2}{\gamma+1}\right) \\ \eta_2 &= M_4(\xi_1,\xi_2)u_1^1 + M_5(\xi_1,\xi_2)c + M_6(\xi_1,\xi_2)c^2 \end{aligned} \tag{1.13}$$

where $M_i\,(\xi_1, \xi_2)$ $(1 \leqslant i \leqslant 6)$ are given functions.

Series (1.11) and their first coefficients (1.13) were used in [15] to describe the three-dimensional escape of an initially homogeneous and stationary gas into a vacuum.

To investigate the properties of the solution of the problem of shockless strong compression of quasi-one-dimensional gas layers in the neighbourhood of the point $t = t_0$, it is convenient to replace (1.11) by the different representation [1]

$$\mathbf{U} = \sum \mathbf{U}_{k,0}(t, \xi_1, \xi_2)(c-1)^k / k! \tag{1.14}$$

2. DETERMINATION OF THE LAW OF MOTION OF THE PISTON

Now that we have a solution of problem (1.1), let us find $r = r_p(t)$—the path of a gas particle travelling from a point $(t = t_0, r = r_0)$ to the sonic line C^+, i.e. $r_0 = t_0 + 1$. This path may be defined as the path of an impermeable piston compressing the gas as stipulated.

Considering the formula (1.2) for transformation to physical space at $t = t_p(t)$ and differentiating both sides with respect to t, we obtain the relation

$$r_p'(t) = \Psi_{ut}(t, r_p'(t)) + \Psi_{uu}(t, r_p'(t))r_p''(t) \tag{2.1}$$

(we have used the condition that the piston is impermeable to the gas: $u(t, r)|_{r = r_p(t)} = r_p'(t)$, the gas velocity at the piston equals the velocity of the piston). Denoting the velocity of the piston by $u = u_p(t)$, we have $r_p'(t) = u_p(t)$. Rewriting (2.1) taking this relation into account, we obtain a problem for determining the velocity of the piston

$$\Psi_{uu}(t, u_p(t))du_p(t)/dt = u_p(t) - \Psi_{ut}(t, u_p(t))$$
$$u_p(t_0) = 0 \tag{2.2}$$

When $\nu = 0$, the solution of problem (2.2), as well as the velocity of sound in the gas and the pressure at the piston, have a finite representation [4]

$$u_p(t) = \frac{2}{\gamma - 1}[\tau^{-\varkappa} - 1], \quad c_p(t) = \tau^{-\varkappa}$$
$$p_p(t) = \frac{1}{\gamma}\tau^{-2\gamma/(\gamma+1)}; \quad \tau = \frac{t}{t_0}, \quad \varkappa = \frac{\gamma - 1}{\gamma + 1} \tag{2.3}$$

When $\nu = 1, 2$, the functions Ψ_{ut} and Ψ_{uu} are given in the neighbourhood of the point ($t = t_0$, $u = 0$) by the series

$$\Psi_{ut}(t, u) = 1 + \sum [Y_k(t) + (t+1)Y_k'(t)](k+2)u^{k+1}/k!$$
$$\Psi_{uu}(t, u) = (t+1)\sum Y_k(t)(k+2)(k+1)u^k/k! \tag{2.4}$$

If $-1 < t_0, < 0$, then $Y_0(t_0) \neq 0$. Therefore, if $t = t_0$, $u = 0$, the condition $\Psi_{uu} \neq 0$ is satisfied. Consequently, the Cauchy problem (2.2) is not posed at a singular point and has a unique solution which is analytic in the neighbourhood of $t = t_0$. Retaining a finite number of terms in the infinite series (2.4), we obtain approximations to the solution. In particular, if we retain terms containing only $Y_0(t)$, then the approximate law of motion $u_0(t)$ for the piston in the neighbourhood of the point $t = t_0$ is given by the following formulae:

for $\nu = 1$

$$u_0(t) = \frac{1}{\sqrt{t+1}}\left[\frac{C_1}{(1-\sqrt{t+1})^{\varkappa}} - \frac{2}{\gamma - 1} + \frac{1-\sqrt{t+1}}{\gamma}\right] \tag{2.5}$$

$$C_1 = (1-\sqrt{r_0})^{\varkappa}\left(\frac{2}{\gamma-1} - \frac{1-\sqrt{r_0}}{\gamma}\right)$$

for $\nu = 2$

$$u_0(t) = \frac{2}{\gamma+1}(t+1)^{-1}[-\ln(t+1)]^{-\varkappa}\int_{t_0}^{t}[-\ln(t+1)]^{-2/(\gamma+1)}dt \tag{2.6}$$

Isolating the principal terms of the functions on the right of this last equality as $t \to t_0$, we obtain the following representation

$$u_0(t) = -\frac{2}{(\gamma+1)r_0 \ln r_0}(t - t_0)\left[1 - \frac{2}{(\gamma+1)r_0 \ln r_0}(t-t_0) + (t-t_0)^2 u_{01}(t)\right] \times$$
$$\times\left[1 - \frac{1}{r_0}(t-t_0) + (t-t_0)^2 u_{02}(t)\right]\left[1 - \frac{(\gamma-1)}{(\gamma+1)r_0 \ln r_0}(t-t_0) + (t-t_0)^2 u_{03}(t)\right]$$

The functions $u_{0i}(t)$ ($i = 1, 2, 3$) are analytic in the neighbourhood of the point $t = t_0$ and correspond to the remainder terms in the expansions of an integral, $(t + 1)^{-1}$ and a power of $-\ln(t + 1)$, respectively. To determine $u_p(t)$ as $t \to -0$, one must use series (1.3).

Then the differential equation of problem (2.2) takes the form

$$\left\{t\left[\sum T_{k+1}''(u)\frac{t^k}{(k+1)!}\right]\frac{du}{dt} = u - \sum T_{k+1}'(u)\frac{t^k}{k!}\right\}\Bigg|_{u=u_p(t)} \tag{2.7}$$

To determine $u = u_p(t)$ taking formulae (1.4) into account, we introduce another unknown function $\upsilon_* = 1 + (\gamma - 1)u_p(t)/2$. In particular, letting $t \to -0$, we conclude from (1.4) that $\upsilon_*(t)/c_p(t) \to 1$. Noting the form of $c_p(t)$ when $\nu = 0$ (see (2.3)), we assume that the function $\nu_*(t)$ for $\nu = 1, 2$ has the form

$$\upsilon_*(t) = A\tau^{-\varkappa}[1 + w_*(\tau)] \tag{2.8}$$

where A is an arbitrary constant, and it is assumed that the new unknown function $w_*(\tau)$ tends to zero as $t \to +0$. Using standard expansions of the functions $(1 + w_*)^\alpha$, $(1 + w_*)^{-1}$, $\ln(1 + w_*)$ in powers of w_*, we obtain instead of (2.7) an equation for $w_*(\tau)$ which, say in the case $1 < \gamma < 5/3$, has the form

$$\frac{dw_*}{d\tau} = B(\nu, d, \gamma, A)\tau^{(1-\alpha)\varkappa}\left[\frac{1 + F_2(\tau, w_*)}{1 + F_1(\tau, w_*)}\right] \tag{2.9}$$

$$F_i(\tau, w_*) = \sum F_{i,k}(\tau, \tau^\delta)\frac{w_*^k}{k!}, \quad \delta = \text{const} > 0, \quad F_i(0,0) = 0, \quad i = 1,2$$

where $B(\nu, d, \gamma, A)$ is a constant which depends on the parameters shown and $d = -t_0 = 1 - r_0$ is the width of the initial gas layer.

Note that if, beginning from some t_1: $t_0 \leqslant t_1 \leqslant 0$, the function $u_p(t)$ lies in the domain (1.5), that is, the convergence domain of series (1.3), then for all $\tau : 0 \leqslant \tau \leqslant \tau_1 = t_1/t_0$ the series defining the functions $F_{1,2}(\tau, w_*)$ will be convergent if w_* is small.

When $\gamma \geqslant 5/3$ one obtains different differential equations, which are too cumbersome to be presented here.

The solution of Eq. (2.9) for $1 < \gamma < 5/3$ and the corresponding equations for other values of $\gamma > 1$ lead to the following formulae

$$\upsilon_*(\tau) = \begin{cases} A\tau^{-\varkappa} + \nu d\dfrac{\gamma^2 - 1}{2(5/3 - \gamma)(3 - \gamma)}A^\alpha\tau^{1/2} + \ldots, & 1 < \gamma < \dfrac{5}{3} \\ A\tau^{-1/4} - \nu d\dfrac{3}{8}A^2\tau^{1/2}\ln\tau + & \\ +\nu dA^2\left(\dfrac{3}{2}\ln A - \dfrac{3}{4}\right)\tau^{1/2} + \ldots, & \gamma = \dfrac{5}{3} \\ A\tau^{-\varkappa} + \nu d\dfrac{\gamma - 1}{(\gamma - 5/3)}A^2\tau^{(3-\gamma)/(\gamma+1)} + \ldots, & \gamma > \dfrac{5}{3}, \ \gamma \neq 3 \\ A\tau^{-1/2} + \nu d\dfrac{3}{2}A^2 + \nu d\dfrac{3}{8}A\tau^{1/2}\ln\tau - & \\ -\nu dA\left(\dfrac{3}{4}\ln A + \dfrac{5}{4}\right)\tau^{1/2} + \ldots, & \gamma = 3 \end{cases} \tag{2.10}$$

where the dots represent terms of higher order infinitesimals than the last of those shown.

It follows from a comparison of the first and last terms in each of the formulae (2.10) that, for τ sufficiently close to zero, $w_*(\tau)$ is indeed small, and therefore the curves defining the laws obtained lie in the convergence domain of series (1.3). This justifies the derivation of the asymptotic laws (2.10).

In particular, if t_0 is sufficiently close to zero, series (1.3) may be used for all t: $t_0 \leqslant t \leqslant 0$. Then representations (2.10) will hold for all τ: $0 \leqslant \tau \leqslant 1$. In that case the arbitrary constant A must be determined from the initial condition of problem (2.2), which, for the function $\upsilon_*(\tau)$, is written as $\upsilon_*(1) = 1$.

In the general case, the constant A is determined from the condition that at a certain time $t = t_2$: $t_0 < t_2 < 0$ functions (2.5), (2.6) and functions (2.10)—as different representations of the function $u_p(t)$—must be suitably matched.

The fact that the exponent in the second terms on the right in (2.10) for $\gamma < 5/3$ is independent of γ, while for $\gamma > 5/3$ there is such a dependence, stems from the fact that the differences $5/3 - \gamma$ and $\alpha - 2$ have the same sign. Consequently, at different γ values, different terms of (1.4) yield the principal parts of the functions $T_2(\nu)$ as $\nu \to +\infty$.

In the case of shockless strong compression of quasi-one-dimensional gas layers, the easiest way to determine the law of external action, taking representation (1.11) into account, is to find $c_p(t, \xi_1, \xi_2)$, namely, the velocity of sound at the compressing piston. In that case the law of motion of the piston in the normal direction is given by

$$\eta_p(t,\xi_1,\xi_2)=[\sum\eta_k(c,\xi_1,\xi_2)t^k/k!]_{c=c_p(t,\xi_1,\xi_2)}$$

Since the piston is impermeable, the following equality must hold on it

$$\frac{\partial\eta_p}{\partial t}=u^1(t,c,\xi_1,\xi_2)\Big|_{c=c_p(t,\xi_1,\xi_2)}$$

Consequently, we obtain the following problem for determining c_p

$$\left[\sum\frac{\partial\eta_k(c_p,\xi_1,\xi_2)}{\partial c_p}\frac{t^k}{k!}\right]\frac{\partial c_p}{\partial t}=\sum[u_k^1(c_p,\xi_1,\xi_2)-\eta_{k+1}(c_p,\xi_1,\xi_2)]\frac{t^k}{k!} \tag{2.11}$$

$$c_p(t_0,\xi_1,\xi_2)=1$$

The differential equation in this problem is actually an ordinary differential equation in which ξ_1 and ξ_2 occur as parameters. Its solutions are also given by formulae (2.10), in which the following substitutions must be made. On the left, $\nu_*(\tau)$ must be replaced by $c_p(\tau, \xi_1, \xi_2)$. On the right, it must be assumed that $A = A(\xi_1, \xi_2)$, and ν should be replaced by $[k_1(\xi_1, \xi_2) + k_2(\xi_1, \xi_2)]$—the sum of the principal curvatures of the surface Γ. The constant d remains unchanged as the width of the initial layer in the direction of the normal to Γ, which is the same for all admissible values of ξ_1 and ξ_2.

Note that as $\nu \to 0$ (or $k_{1,2}(\xi_1, \xi_2) \to 0$) formulae (2.10) approach the exact formulae (2.3) for the plane-symmetric case.

In the compression of quasi-one-dimensional layers not from within but from the outside, formulae (2.10) remains unchanged, but then

$$\upsilon_*=1-\frac{\gamma-1}{2}u_p$$

Of course, relations (2.5), (2.6) and (2.10) obtained here may be refined by using more terms from the infinite series that define the solution of problem (1.1) in its various forms.

An analysis of formulae (2.10) indicates certain considerations with regard to feasible physical experiments.

First, formulae (2.10) provide yet another confirmation of the well-known conclusion concerning easily compressible media (small γ) and media compressible only with difficulty (large γ): when $\gamma < 3$, the second terms on the right of formulae (2.10) tend to zero as $\tau \to 0$, while for $\gamma > 3$ they tend to infinity.

Second, in all easily compressible media, if $\gamma < 5/3$, the second term has an exponent that is independent of γ.

And, surely the most important: for $\gamma < 3$, the additional external energy outlay involved in the transition from the compression of plane layers to that of quasi-one-dimensional layers is finite.

It also follows from formulae (2.10) that the essential difference in the external forces that produce strong compression of different gas layers will manifest itself in the directions in which they are applied: along the normals to the different initial surfaces.

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